# Inverse Problems for Integro-differential Operators

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To our families and all who have supported us

# Contents

# Preface

Inverse problems are concerned with determining causes by knowledge of consequences. They lie at the heart of scientific inquiry and technological development and remain a central topic to mathematical sciences. The mathematical research of inverse problems has its own philosophy and methodologies. This book is devoted to a frontier topic on the inverse problems for integro-differential operators, a.k.a. nonlocal operators. Due to its theoretical particularity and practical significance, the corresponding study has received considerable attention and growing interest in the inverse problems community. Hence, it is a timely moment for a research monograph devoted to this intriguing and important field of mathematical research.

It is known the nonlocality may occur in time or space or both. This book is mainly concerned with the nonlocality in space. It covers the fundamental aspects of both the forward and inverse problems associated with integro-differential operators. For the forward problems, we introduce several useful properties, such as the well-posedness, maximum principles and unique continuation property. For the inverse problems, we cover the modelling, unique identifiability and stability issues as well as reconstruction methods for a variety of nonlocal inverse problems and connect them to physical applications. There are some pioneering contributions as well as growing results in the literature in this field.

On the one hand, we summarize and review the pioneering developments in a systematic and comprehensive manner. On the other hand, we present overviews on some new developments, especially those by the two authors as well as their coauthors. This field is still being under fast development. It is replenished with challenging problems, and moreover new applications keep giving rise to new nonlocal inverse problems. It is our aim to introduce a frontier field of research as well as to inspire further developments with novel perspectives and new insights. This book can serve as a textbook for graduate students or beginning researchers who are interested in this active field of research, and it can also serve as a handy reference for active researchers.

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